LINC

Learning about Interacting Networks in Climate

**Marie Curie Initial Training Networks (ITN)**

FP7- PEOPLE - 2011- ITN

**Grant Agreement No. 289447**

WorkPackage WP5: *Tipping points of the climate system*

# Deliverable D5.2

# Report on indicators of tipping points in terms of network quantities

Emilio Hernández-García, UIB

Giulio Tirabassi, UPC

Cristina Masoller, UPC

Henk A. Dijkstra, UU

Release date: 30 May 2014

Status: PU

## EXECUTIVE SUMMARY

The LINC project aims to provide training and perform research on the characterization of climatic processes in terms of networks or graphs. The objective of WP5 ``Tipping Points in the Climate System'' is to develop network indicators of regime shifts that could be used to analyze states and predict evolution of the climate system close to tipping points. The subject of the present deliverable is to summarize the network techniques that have been developed so far within the consortium to identify and to anticipate climatic transitions and the proximity to them from climatic time series.

Here we compile different techniques based on the topological characterization of correlation networks obtained from climatic time series. We analize degree, assortativity and clustering distributions, and show how they change when a climatic system is approaching a critical transition. The approaches are illustrated by analyzing spatio-temporal time series obtained from a model of desertification under climatic change (modelled as a drift in a rainfall intensity parameter). The quality of the different indicators is estimated, and compared between them and with classical (i.e. non-network based) techniques to anticipate tipping points. The performance of the network indicators turns out to be superior to that of the classical ones, being the variance of the degree distribution and different aspects of the assortativity distribution especially suited to provide early warnings of the proximity to a climatic shift.

Besides WP5, this deliverable is relevant for WP1 and WP4, since the concept of tipping point and the associated climatic transition is strongly related to climate change and its prediction.

# Deliverable Identification Sheet

| | |
|---|---|
| **Grant Agreement No.** | **PITN-GA-2011-289447** |
| **Acronym** | **LINC** |
| **Full title** | **Learning about Interacting Networks in Climate** |
| **Project URL** | http://climatelinc.eu/ |
| **EU Project Officer** | Emanuela MARINELLI |

| | |
|---|---|
| **Deliverable** | **D5.2 Report on indicators of tipping points in terms of network quantities** |
| **Work package** | **WP5 Tipping points of the climate system** |

| | | | | |
|---|---|---|---|---|
| **Date of delivery** | **Contractual** | M 30 | **Actual** | 30-May-2014 |
| **Status** | version. 1.00 | | final ☑ draft ☐ | |
| **Nature** | Prototype ☐ Report ☑ Dissemination ☐ | | | |
| **Dissemination Level** | Public √ Consortium ☐ | | | |



| | | | | |
|---|---|---|---|---|
| **Authors (Partner)** | Emilio Hernández-García (UIB), Giulio Tirabassi (UPC), Cristina Masoller (UPC), Henk A. Dijkstra (UU) | | | |
| **Responsible Author** | **Emilio Hernandez-García** | | **Email** | emilio@ifisc.uib-csic.es |
| | **Partner** | UIB | **Phone** | +34 971171307 |





| | | |
|---|---|---|
| **Abstract (for dissemination)** | This deliverable evaluates the performance of several indicators, based in network theory, suitable to provide early warnings of the approach to a climatic abrupt change, using data from spatio-temporal time-series. Network indicators turn out to be more efficient than classical ones. | |
| **Keywords** | Tipping points, early warnings, regime shifts, network theory | |



| **Version Log** | | | |
|---|---|---|---|
| **Issue Date** | **Rev No.** | **Author** | **Change(s)** |
| 30-05-2014 | 001 | E. Hernández-García | Final document |

# Contents

# 1 Introduction

Paleodata reveal that abrupt changes in the Earth's climate system have occurred in the past. Estimated present-day climate trends point towards important climatic changes occurring in the near future, largely influenced by anthropogenic actions. Since many of the studied paleoclimatic changes occurred in a rather fast (on geological scales) manner associated to the reach of a *tipping point*, it is important to develop tools able to predict and anticipate the possible abrupt changes that can occur in our future. To develop such techniques, in the context of applications of network theory, is one of the tasks of the LINC training network and explicitly addressed in Workpackage 5.

The first results in this topic by members of the LINC consortium have already appeared and applied in the context of the Atlantic Meridional Overturning Circulation rapid change (Van Der Mheen et al., 2013). These applications and further developments will be the subject of Deliverable 5.4. Here we compile the methodological part of these early developments as well as more recent network-theory quantifiers of abrupt shifts, and illustrate them with applications to a relatively simple model of desertification transitions. This type of climatic shift has already occurred in the past and may affect in the future large regions of the Earth if present-day trends persist.

We briefly summarize in Section 2 previous approaches to climatic transition detection, based in classical non-network techniques. Our new network indicators are presented in Sect. 3, as applied to a desertification transition model. The quality of the different indicators is compared. The final section 5 contains our conclusions.

# 2 Previous approaches

Lenton et al. (2008) studied which variables in the climate system can be susceptible to reach and pass certain critical threshold in the value of some control parameters and produce an abrupt change in the state of the system. Usually these critical points are called tipping points, and the system variables that have these kind of critical points are called tipping elements. A subsystem $\Sigma$ is a tipping element of the system if $i)$ the parameters controlling the system can be transparently combined into a single control $\rho$ $ii)$ exist a critical control value $\rho_{crit}$ from which any significant variation by $\delta\rho > 0$ leads to a qualitative change $(\hat{F})$. They circumscribe the definition of tipping element to human activities. Then, besides the above mentioned conditions, they need to $iii)$ establish a pertinent time to study the phenomena, that include the political time horizon $T_p$, the critical time $T_c$ in which the transition could happen, and the ethical time horizon $T_E$ beyond any change in the future influencing today's decisions. Finally, it is also important to consider $iv)$ the significant population in the tipping element $\Sigma$. Using the aforementioned definitions, the authors studied nine tipping element in the earth systems, and find that at least in Arctic sea-ice and the Greenland ice sheet could reach their critical point within this century under anthropogenic climate change. Two fundamental questions arise here: can we stay clear of $\rho_{crit}$? can $\hat{F}$ be tolerated? (Lenton, 2011), Early warnings in the tipping elements defined previously can be possible in principle, at least at a level that could provide useful information to help manage the risks that they pose. The authors classified the indicators in different systems and compared the measure and the expected evolution.

In order to study the tipping phenomena in the climate system, Thompson and Sieber

(2011) suggest to split the whole system in its more important elements: atmosphere, ocean, land, ice, and biosphere. These elements have their own internal coupling and internal potential. They are also exposed to a external forcing provided by sunlight and cosmic rays. This means that the climate system is an open dissipative system. It losses its inner energy through these interactions. In the context of bifurcation theory, the climate system would have a *codimension-1* bifurcation when a slow variation of the parameter push the system through the critical point to an alternative state. Due to our interest on abrupt changes in the climate in the Earth, a classification of bifurcations according to their repercussion, as done by Thompson and Sieber (2011). They established a three different types of bifurcations: *safe, explosive, and dangerous*. Abrupt changes correspond to dangerous and explosive bifurcations. It was found that the local decay rate (LDR) of pertubations tends to zero in most of these transitions. This is a consequence of the Critical Slowing Down (CSD) property of all local bifurcations (Thompson and Sieber, 2011). The principal limitation of LDR is that can not distinguish the kind of bifurcation that might be caused by a high noise present in the forcing (*flickering*).

A test bed for the study of climate transitions is provided by geological records. In (Dakos et al., 2008) the autocorrelation in times series from geological record of eight ancient events (end of greenhouse Earth, end of glaciation IV, end of glaciation III, end of glaciation II, end of glaciation I, Bolling/Allerod transition, end of Youger Dryas and desertification of N. Africa) was calculated. In all of these cases, the autocorrelation showed an increased in the period before the transition occurred. It is important to keep in mind that autocorrelation is a consequence of the CSD behavior; which appears when the system is moving gradually toward a bifurcation point. Therefore, for transitions caused by a sudden large change in the control parameter so that the system would have no time to loss gradually resilience, the slowing down may not be a sign.

Since the main problem with the paleo-records is the quality of the data, another possible way to understand transition in dynamical system is through simulation. Several works (Dakos et al., 2009; Donangelo et al., 2010; Lade and Gross, 2012) have studied the behavior close to a transition and in all of them an increase in the value of spatial correlation was found, as in the temporal autocorrelation at lag-1. Studies where the data come from experimental realizations are however scarce. Dakos et al. (2009) studied two-dimensional heterogeneous ecology models related to climatic change. They included logistically growing resource, eutrophication and vegetation-turbidity models. Two types of correlation function were calculated: the spatial correlation at high distances and the autocorrelation at lag-1. Also, different scenarios of connectivity and heterogeneity were analyzed. In the simplified scenario, when the connectivity between two spatial cells with different carrying capacities is high, the shift happened in the same value of control parameter in both cells. Instead, when the connectivity was low, different values of harvesting rate are necessary to move the cells through the bifurcation. In this simplified scenario with only two cells, the recovery rate was also measured in each cell. Indeed, close to transition the recovery rate was slower than far from transition. To study the collective behavior of the system close to a transition, the complete spatial models were considered. In a wide range of conditions for all models the spatial correlation between cells increased. More specifically, when the coupling between cells was low the spatial correlation between them remains almost constant. As a consequence, the shift of the whole system is gradual, as each cell shifts almost independently. Instead, when the connectivity was high, the change in the state of the system is abrupt as a consequence of the increased of the spatial correlation. A related model was studied by Donangelo et al. (2010). They considered the ”spatial version of the mean field model of a lake” represented by a stochastic partial differential equation for the field state of the lake. The field associated with the state of the lake exhibits a contrasting change between two stable solutions. The aim of their work is characterizing this transition in terms of macroscopic quantities, particularly: $i$) Spatial variance of the field. $ii$) Autocorrelation at some point of the space $iii$) Cluster structure, that means studying what happens with the neighborhood of some cell experiences a change in its state, and $iv$) Spatial correlation of the field corresponding to a two different cells separated by a determined distance. Searching for early warning in this system, the authors found that both, spatial variance and temporal variance of the field start to increase as the system approaches to a bifurcation point in the parameter space, in fact, ”spatial variance raise earlier than temporal variance” although both have a maximum exactly when the system changes to an alternative state. Another early warning in the system is pattern formation: before the system reaches the alternative state each component starts to loose resilience, the repercussion of that is an increasing fluctuation around the equilibrium point in the state of each cell and therefore a trend to become similar to its neighborhood. The control parameter has a stochastic component, that could explain why some cell remains in the previous state and the other one shifts.

# 3 Network based indicators

A novel perspective for finding leading indicators of precursors in dynamical systems, is the use of functional network coming from spatial correlation between the units that compose the system. In this approach, the main idea is to acquire knowledge by studying the topology of the functional network, defining new indicators of critical transition in those terms. In this context, Van Der Mheen et al. (2013) considered a two-dimensional (meridional-depth) model of the Atlantic MOC. This model exhibits a tipping point at certain value of the fresh water control parameter. A functional network was reconstructed from each temperature field time series and analyzed in terms of the degree distribution, and clustering coefficient. New network-based indicators were found associated with the spatial correlation of the temperature field. The main results are: i) the degree of the functional network increase close to the bifurcation ii) as the system approaches to the tipping point, a hub starts to appear meaning that the ocean is more correlated in certain region.

In the following we present these quantifiers, as well as newly developed ones, and apply them to spatio-temporal time series coming from a model of desertification which is relevant to understand how climate change will affect vegetation cover.

# 4 Network indicators of climatic shifts: the example of desertification transitions

The threat of desertification is one of the most impelling problem for resources planification and international politics. Something around the $40\%$ of Earth’s land is covered by drylands. More than two billion people is settled in these regions and among the $90\%$ of them live in developing countries. It can be estimated that almost 1 billion people are under the severe threat of desertification. It appears critical then to understand better this phenomenon, and especially how to anticipate its occurrence. In this section we will give an overview of the problem under the dynamical system and network point of view.

## 4.1 Desertification as a catastrophic transition

It has been proposed that ecological systems might be, in some parameters' range, a bistable system affected by hysteresis. In other words, provided the system a certain amount of water, it can develop either a highly vegetated state or a dry state, with almost no vegetation, depending on the history of the provided water. This idea can be easily formalized through a dynamical model presenting a saddle node transition. In such a model, a highly vegetated stable branch disappears once the rainfall is sufficiently decreased, and it the system collapse on a poorly vegetated state. From this state, increasing the rainfall does not allow the system to come back to the previous highly vegetated state, unless another bifurcation takes place.

A model displaying these characteristics could be a local positive feedback (LPF) model as (Dakos et al., 2011):

$$dw_t = (R - \frac{w}{\tau_w} - \Lambda w B + D\nabla^2 w)\, dt + \sigma_w w_0 dW_t\ , \tag{1a}$$

$$dB_t = \left(\rho B \left(\frac{w}{w_0} - \frac{B}{B_c}\right) - \mu \frac{B}{B + B_O} + D\nabla^2 B\right) dt + \tag{1b}$$

$$+ \sigma_B B_0 dW_t\ , \tag{1c}$$

where $w$ (in mm) is the soil water amount and $B$ (in g/m$^2$) is the vegetation biomass. The quantity $D$ is the diffusivity and $\tau_w$, $\mu$, $\rho$, $\Lambda$, $w_0$, $B_O$, $B_c$ are additional constants explained in Table 1. Finally, $R$ is the amount of rainfall which is used as the bifurcation parameter of the system. We assume it is the parameter that climate change will affect. Additive Gaussian white noise is prescribed with amplitudes $\sigma_w$ and $\sigma_B$ for soil water and biomass, respectively. For the choice of parameters presented in Table 1 the system presents bistability, and the associated bifurcation diagram for the homogeneous states is presented in Fig. 1. The deterministic homogeneous solutions of the LPF model and their linear stability can be determined analytically. For all values of $R$, the trivial solution ($B = 0$, $w = \tau_w R$) exists. For the standard parameter values shown Table 1, the trivial solution is linearly stable for $R < 2$ mm/day and unstable for $R > 2$ mm/day (see Fig. 1). At $R = 2$ mm/day, a transcritical bifurcation occurs and two additional branches of steady solutions emerge. Solutions on the lower branch are not considered here because they have $B < 0$, i.e., they are physically non-realistic. Solutions on the upper branch are unstable for values of $R$ down to $R_c = 1.067$ mm/day. At this $R$-value a saddle-node bifurcation occurs which provides a linearly stable upper branch of solutions for $R > 1.067$ mm/day. Finally, a fourth homogeneous solution exists but it has also values of $B < 0$ for every $R$ value and hence is not further considered here.

In order to determine inhomogeneous vegetation patterns in the stochastic case, the model equations (1) are numerically solved on a periodic square grid composed of $100 \times 100 = 10^4$ grid cells on a regular lattice with dimension $L = 100$ m. The evaluated model data consists of a set of time series (500 time steps with $\Delta t = 0.01$ days) of statistically equilibrated biomass fields $B$ for different fixed rainfall parameters $R$. Time series related

Table 1: Parameters of the local positive feedback model (LPF) given by Eq. (1).

| Parameter | Meaning | Value |
|---|---|---|
| $D$ | Exchange rate | 0.5 $m^2 day^{-1}$ |
| $\Lambda$ | Water consumption rate by vegetation | 0.12 $m^2 g^{-1}$ $day^{-1}$ |
| $\rho$ | Maximum vegetation growth rate | 1 $day^{-1}$ |
| $B_c$ | Vegetation carrying capacity | 10 g $m^{-2}$ |
| $\mu$ | Maximum grazing rate | 2 g $day^{-1}$ $m^{-2}$ |
| $B_O$ | Half-saturation constant of vegetation consumption | 1 g $m^{-2}$ |
| $\sigma_w$ | Standard deviation of white noise in water moisture | 0.1 |
| $\sigma_B$ | Standard deviation of white noise in vegetation biomass | 0.25 |
| $w_0$ | Water moisture scale value | 1 mm |
| $B_0$ | Biomass density scale value | 1 g $m^{-2}$ |
| $\tau_w$ | Water moisture scale time | 1 day |

to 10 different values of $R$ with $1.1 \leq R \leq 1.8$ mm/day are analyzed. For $R < R_c$ only the desert-like solution, with $B = 0$ over the whole domain, is found.

## 4.2 Classical indicators

The most prominent statistical measure to infer critical slowing down (CSD) from data is the lag-1 autocorrelation. The spatially averaged autocorrelation of the LPF model data is shown in Fig. 2a for each value of $R$. As $R$ decreases and approaches $R_c$, the system indeed experiences an increase in autocorrelation, a distinct fingerprint of CSD.

It has also been suggested that spatial statistics may be used to detect CSD (Dakos et al., 2011). In particular, an increase in spatial correlation of the system is expected when the system experiences CSD close to the transition. A typical measure of spatial correlation that has been used is Moran's coefficient, $I$, defined here through the biomass mass field $B$ as

$$I \equiv \frac{N}{\sum_{ij} g_{ij}} \frac{\sum_{ij} g_{ij}(B_i - \bar{B})(B_j - \bar{B})}{\sum_i (B_i - \bar{B})^2} , \tag{2}$$

where $g_{ij} = 1$ if $i$ and $j$ are two adjacent grid cells and $g_{ij} = 0$ otherwise. The spatially averaged biomass is indicated by $\bar{B}$. Moran's coefficient $I$ is shown in Fig. 2b and demonstrates that the spatial correlation increases in a similar way – although stronger – to the temporal correlation as $R$ decreases and approaches $R_c$.

However, despite the fact that the classical indicators are able to reflect CSD, they change in a very smooth, gradual and monotonic way. From a strictly local point of view, that is, based on a few closely spaced $R$ values, it is not possible to estimate the proximity of the system to $R_c$. In other words, the correlation coefficients suffer from a lack of distinct features necessary to provide a pronounced early-warning signal of desertification.

## 4.3 Network Analysis

The correlation network of the biomass data is given by the following adjacency matrix,

$$A_{ij} = \mathrm{H}\Big(|\mathcal{C}(B_i, B_j)| - \theta\Big) , \tag{3}$$

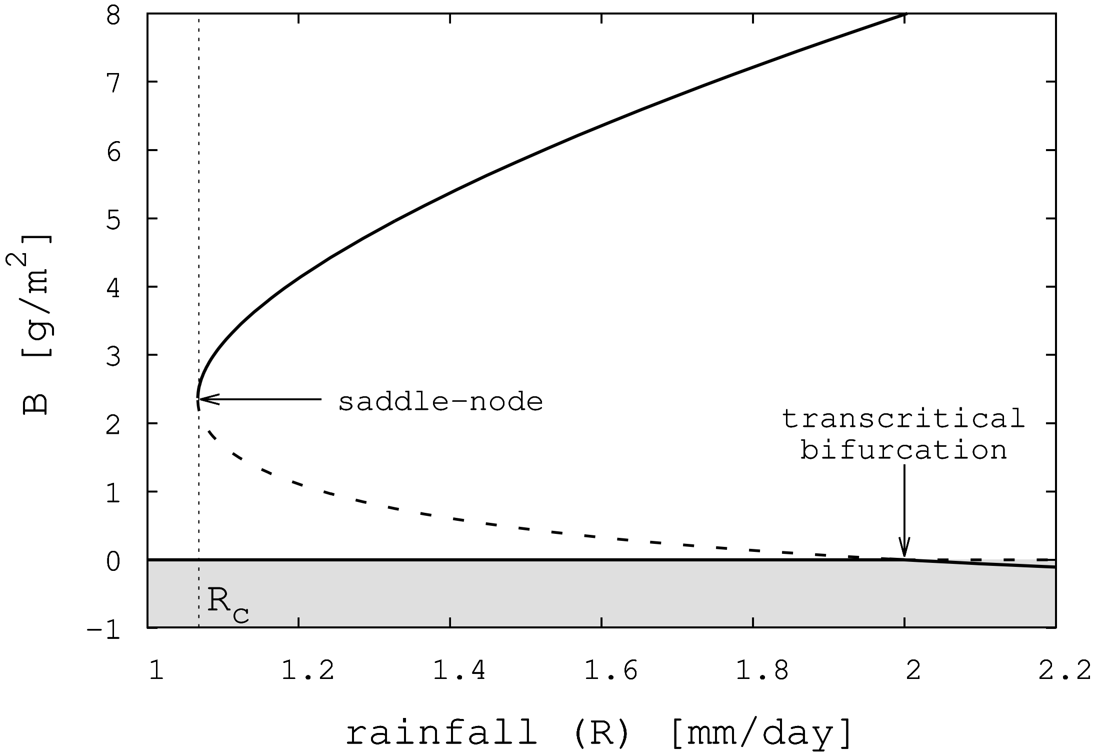
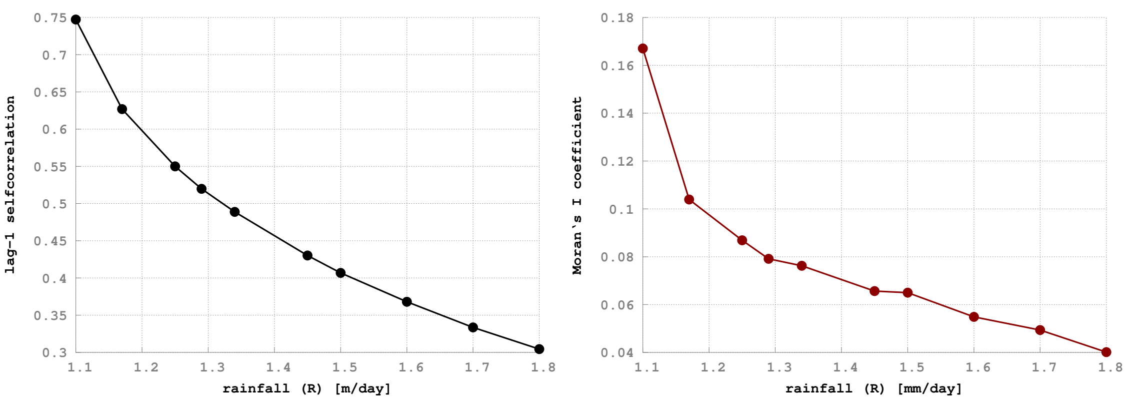
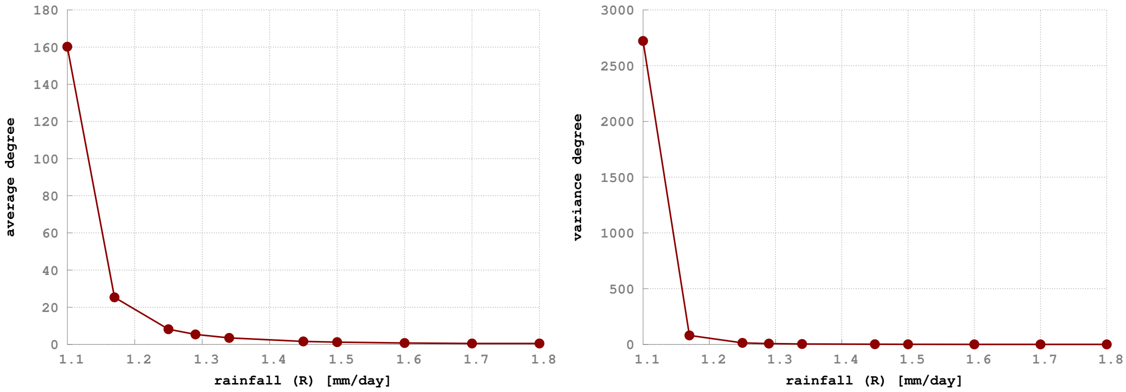



Figure 1: Bifurcation diagram of the local positive feedback model (LPF) given by Eq. (1). Curves depict the biomass ($B$) at steady homogeneous states, that is, determined under vanishing diffusion and noise. Linearly stable branches are denoted by solid lines, whereas linearly unstable branches are indicated by dashed lines. The unvegetated desert state is the only homogeneous state existing to the left of $R_c$, and it coexists with a vegetated state between the saddle-node and the transcritical bifurcation points. The shaded region indicates to the non-physical negative $B$ values.




(a) (b)

Figure 2: (a) Spatially averaged lag-1 autocorrelation and (b) Moran's coefficient $I$ computed using the spatial biomass distribution at the last time-step of the simulation for different values of the rainfall parameter $R$.




(a) mean (b) variance

Figure 3: (a) Mean and (b) variance of network degree distribution as function of the bifurcation parameter $R$.

where H is the Heaviside step function, and $\theta$ is a constant threshold indicating statistical significance of the cross-correlation $\mathcal{C}(B_i, B_j)$.

To determine the value of $\theta$ we build the following test variable for the Student's t-test, i.e.,

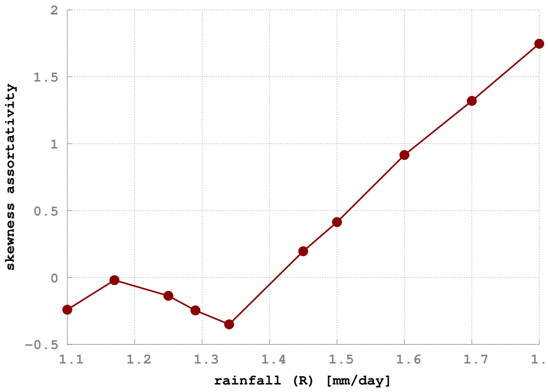
$$t = \frac{\theta}{\sqrt{1-\theta^2}} \sqrt{N_{\text{time-steps}} \frac{1-r}{1+r}} \,, \tag{4}$$

with the null hypothesis $\theta = 0$. Here $r = r(R)$ and $N_{\text{time-steps}}$ are the autocorrelation and the length of the time series, respectively. The test variable takes the effective number of degrees of freedom of the time series into account. From this we can compute the value of $\theta$ which ensures statistical significance of correlations larger than $\theta$. A value of $\theta = 0.2$ guarantees that, for each value of $R$, the zero-lag correlation between linked nodes is statistically significant with a $p$-value smaller than $0.05$ and this value is taken in all results below.

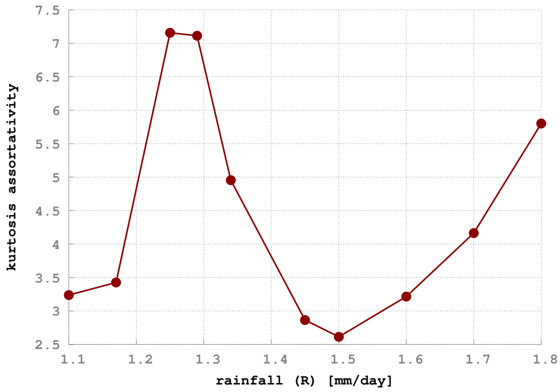
After the construction of the interaction network of the biomass data, we can study changes in the topology of the network due to varying $R$. The most basic characteristic of a network is its **degree** distribution.

Figure 3a shows the mean of the degree distribution as a function of $R$. This network measure is highly sensitive to $R$ near criticality, showing a steep increase close to the transition. Additionally, Fig. 3b shows the variance of the degree distribution as a function of $R$. The increase in variance when approaching the transition point is even more abrupt. The behavior of the average degree is directly related to high degree values occurring near the saddle-node bifurcation. Close to the transition, the vegetation variability synchronizes over the domain, which can be seen as the spatial expression of CSD. This apparent synchronization produces an increasing number of connections approaching the tipping point.

Another basic network measure is the **assortativity** $a_i$ of a node $i$, which is the average degree of its neighbours, that is, of all the nodes to which node $i$ is linked to. In general, the assortativity coefficient of a node can be computed from the adjacency matrix via



(a) assortativity skewness



(b) assortativity kurtosis

Figure 4: (a) Skewness and (b) kurtosis of the assortativity distribution for different $R$.

$$a_i \equiv \frac{1}{k_i} \sum_{j=1}^{N} A_{ij} k_j \ . \tag{5}$$

where $k_i$ is the degree of node $i$. The assortativity characterizes the tendency of a node to be connected to nodes with high degree. Note that a node can have low degree but at the same time high assortativity. As the degree, the assortativity coefficient can range from 0 to $N-1$.

Figure 4a-b shows the skewness and kurtosis of the assortativity distribution[1] for different $R$. The switch in sign of the skewness (Fig. 4a) is a feature that could be related to the forthcoming transition. This switch in sign is unique (a distinct qualitative feature), and it is not prone to false alarms. Furthermore, the kurtosis (Fig. 4b) shows a huge and quick drop just *before* the transition, providing a distinct warning related to the upcoming transition point, especially if considered together with the behavior of the skewness. However, the combined analysis of these two quantities reveals a more pronounced change in the assortativity distribution: Just before the transition the skewness approaches 0, while the kurtosis is close to 3; thus the assortativity distribution is close to a Gaussian distribution near $R_c$. This "Gaussanization" may therefore be used as an early-warning signal of the transition point.

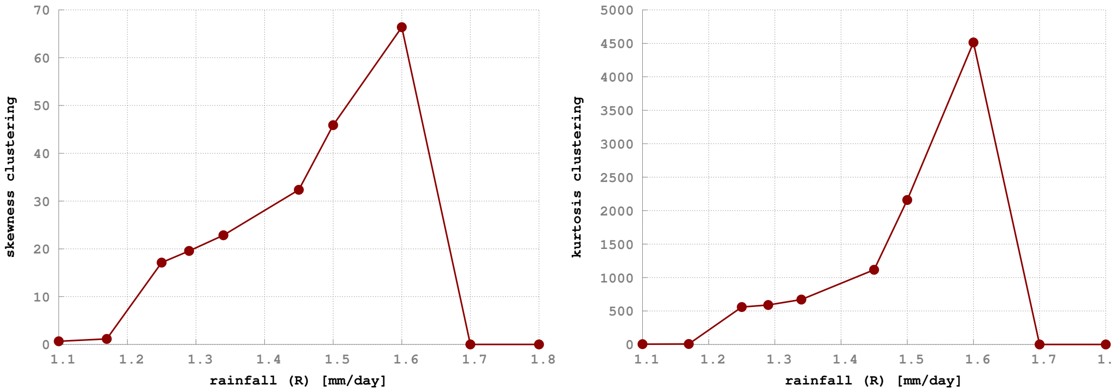
As a third basic network measure we consider the **clustering coefficient** $c_i$.

the skewness and kurtosis of the clustering distribution, shown in Fig. 5c-d, can be used as an early-warning indicator if the two quantities are monitored together. They are displaying a "Gaussanization" of the clustering distribution, similar to the assortativity distribution when approaching the tipping point.

The critical normalization of clustering and assortativity can be quantified numerically through the Kullback–Leibler Distance (KLD), also called relative entropy, which measures the distance between two PDFs. Given two one-dimensional distributions $P(x)$ and $Z(x)$, their relative entropy is defined as

$$KLD \equiv \int_{-\infty}^{\infty} \ln\left(\frac{P(x)}{Z(x)}\right) P(x)\,\mathrm{d}x \ . \tag{6}$$

[1] We exclude the mean and variance of the assortativity distribution from the presentation because they are directly connected to the mean and variance of the degree distribution.




(a) clustering skewness (b) clustering kurtosis

Figure 5: (a) Skewness and (b) kurtosis of the clustering distribution for different $R$.

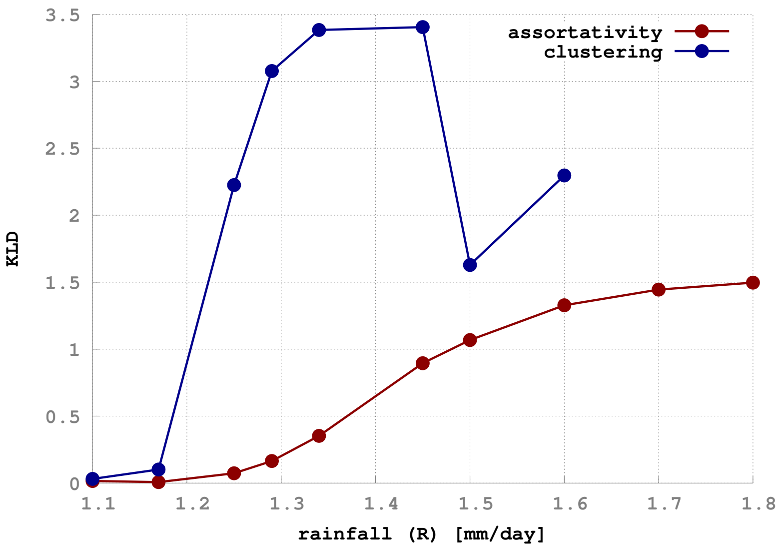



Figure 6: KLD values for clustering and assortativity PDFs with respect to Gaussian distributions with same mean and variance.

Measuring KLD between assortativity/clustering distributions and Gaussians with the same mean and variance allows to quantify the "Gaussanization" of these PDFs when the tipping point is approached. We note that the Gaussanization might a model-specific feature and further analysis is required to assess the generality of this result.

The KLD measure for both assortativity and clustering distributions is plotted in Fig. 6. Obviously, KLD quickly drops to zero when approaching the transition point for both the assortativity and clustering distributions. Small KLD means that the distribution under examination is comparable to a Gaussian, whereas for large values it deviates from Gaussianity. Unlike in the previous cases, the absolute value of the indicator is more important than its derivative.

Table 2: Quality values of different early-warning indicators.

| **Indicator** $J$ | **Class** | **Type** | $Q_J$ |
|---|---|---|---|
| Average degree | Scalar | Network | 0.963 |
| Variance of degree | Scalar | Network | 0.996 |
| Variance of assortativity | Scalar | Network | 0.971 |
| Average clustering | Scalar | Network | 0.904 |
| Assortativity Gaussianity | Distribution | Network | 0.973 |
| Clustering Gaussianity | Distribution | Network | 0.951 |
| Lag - 1 autocorrelation | Scalar | Classical | 0.468 |
| Spatial correlation | Scalar | Classical | 0.730 |

## 4.4 Quality assessment

We have introduced two types of indicators in the previous Section. Some of them are *scalar* quantities in the sense that they refer to single statistical properties characterizing network topology (Average degree, variance of degree, variance of assortativity, average clustering, different types of skewness and kurtosis, ...). On the other hand, the KLD is a *distribution* indicator, since it quantifies properties of a full probability distribution. For both indicator types a quality measure can be defined. First, we define an $\epsilon$-environment around the bifurcation point by all the $R$ values for which $(R - R_c)/R_c < 0.1$. For a scalar-based indicator $J$, we then define the normalized quality measure by

$$Q_J^s = \frac{\langle J' \rangle_{R<\epsilon} - \langle J' \rangle_{R>\epsilon}}{\langle J' \rangle_{R<\epsilon} + \langle J' \rangle_{R>\epsilon}} \,, \tag{7}$$

where the brackets indicate the mean over the interval indicated, and the primes are derivatives with respect to $R$. In this way we achieve that if $J$ shows an abrupt change in its derivative close to the transition then we have $Q_J \approx 1$. In contrast, if the change of $J$ is merely linear when approaching the tipping point then we obtain $Q_J \approx 0$.

In the case of distribution-based indicators we can define a similar quality measure $Q_J^d$ by taking into account $J$ itself instead of its derivative, that is,

$$Q_J^d = \frac{\langle J \rangle_{R<\epsilon} - \langle J \rangle_{R>\epsilon}}{\langle J \rangle_{R<\epsilon} + \langle J \rangle_{R>\epsilon}} \,. \tag{8}$$

Table 2 displays the quality values for both network-based and classical indicators. Obviously, the network-based measures have significantly higher early-warning quality than the classical measures.

# 5 Summary and discussion

In this report we have summarized techniques to anticipate climatic transitions using novel interaction network methods, and exemplify them in a simple vegetation model of desertification transitions. Interaction networks are constructed from time series of biomass fields and the topological changes in the resulting networks are studied along a gradient of decreasing rainfall. We find that network measures like degree, assortativity and clustering may offer novel indicators for identifying an upcoming desertification in semi-arid ecosystems.

Our results are consistent with previous studies that have used network measures as early-warning indicators of critical transitions in models. For example, Van Der Mheen et al. (2013) used an interaction network approach to obtain an early-warning signal of the Atlantic Meridional Overturning Circulation collapse. In that study the network is built using temperature time series, and the behavior of the average degree is monitored as function of freshwater input. Similar to the results in this study, the average degree increases sharply approaching the transition. In contrast, Viebahn and Dijkstra (2014) analyzed the flow field of the wind-driven ocean circulation introducing a flux-based network approach. Also in that context, the degree of the network increases while the system approaches the transition, but a more precise early-warning indicator is given by the network's closeness which shows a big drop near the tipping point due to a local regime change in the flow field.

Here we have introduced measures to assess the quality of different early-warning indicators. Using these quality measures we compared the performance of the novel network-based indicators with the classical indicators based on variance and autocorrelation. We find that the scalar network-based indicators have a higher quality value than the classical indicators. Moreover, distribution-based indicators, here calculated from the assortativity and clustering distributions, have also a high quality value. When these distributions become close to Gaussian, there is an early-warning indication of the upcoming transition. Although these observations hint that the indicators we developed here may offer a strong indirect measure of proximity to critical transitions, they may still be prone to similar limitations that classical indicators face, like producing false positives (Boettiger and Hastings, 2012), or requiring a lot of information for their practical application (Dakos et al., 2012).

Regarding the possibility of false positives, it is well known that CSD is a characteristic feature of transitions related to an eigenvalue going to zero. However, not all the transitions accompanied by an eigenvalue going to zero are also catastrophic that is implying an abrupt discontinuity in the stable branches (Kéfi et al., 2007; Dijkstra, 2011). Thus, CSD can be prone to false alarms, and it will be interesting to test, in future work, the new network indicators against this possible pitfall.

Regarding the sample size of the data, instead, it is clear that the network construction requires a sufficient spatial sampling which temporally based classical indicators do not need. As a test, we computed all the indicators presented in this report also for other two networks, obtained by spatially down-sampling the original $100 \times 100$ dataset considered here. In particular we considered $10 \times 10$ and $50 \times 50$ sub-sets. We find that the scalar-based indicators still perform quite well, whereas the distribution-based indicators do not. In particular, the assortativity Gaussianity still anticipates the transition in the $50 \times 50$ data set, whereas the clustering measure is unable to detect the transition in both data sets. This is a reasonable feature of the proposed distribution-based indicators; in fact a good estimation of a PDF requires a big amount of data, much more than simple low-order moments estimators, as the one used here to build the scalar-based indicators.

While the average degree behavior as a function of $R$ can be easily linked to the spatial expression of the enhancement of correlations of the time series between the nodes, the interpretation of the other indicators, especially that of the distribution-based indicators, is not straightforward. Nevertheless, the network-based indicators seem to offer a better measure of proximity to the tipping point than the classical ones. This may be attributed to the thresholding of the correlation matrix when constructing the interaction network.

This coarse-graining of the information eliminates most of the noise, producing a better signal to noise ratio of the spatial signature of CSD.

The network-based indicators hence offer a promising alternative to detect critical transitions. Further analysis and applications to additional climatic-shift models and empirical time-series will be pursued within the LINC consortium.

# References

Carl Boettiger and Alan Hastings. Early warning signals and the prosecutor's fallacy. *Proceedings of the Royal Society of London B: Biological Sciences* 279, 4734–4739 (2012).

Vasilis Dakos, Stephen R Carpenter, William a Brock, Aaron M Ellison, Vishwesha Guttal, Anthony R Ives, Sonia Kéfi, Valerie Livina, David a Seekell, Egbert H van Nes, and Marten Scheffer. Methods for detecting early warnings of critical transitions in time series illustrated using simulated ecological data. *PloS One* 7(7), e41010 (2012).

Vasilis Dakos, Sonia Kéfi, Max Rietkerk, Egbert H. van Nes, and Marten Scheffer. Slowing down in spatially patterned ecosystems at the brink of collapse. *The American Naturalist* 177(6), 154–166 (2011).

Vasilis Dakos, Marten Scheffer, Egbert H van Nes, Victor Brovkin, Vladimir Petoukhov, and Hermann Held. Slowing down as an early warning signal for abrupt climate change. *Proceedings of the National Academy of Sciences of the United States of America* 105(38), 14308–14312 (2008).

Vasilis Dakos, Egbert H. van Nes, Raúl Donangelo, Hugo Fort, and Marten Scheffer. Spatial correlation as leading indicator of catastrophic shifts. *Theoretical Ecology* 3(3), 163–174 (2009).

Henk A. Dijkstra. Vegetation pattern formation in a semi-arid climate. *International Journal of Bifurcation and Chaos* 21(12), 3497–3509 (2011).

Raul Donangelo, Hugo Fort, Vasilis Dakos, Marten Scheffer, and Egbert H. van Nes. Early warnings for catastrophic shifts in ecosystems: Comparison between spatial and temporal indicators. *International Journal of Bifurcation and Chaos* 20(02), 315–321 (2010).

Sonia Kéfi, Max Rietkerk, Concepción L. Alados, Yolanda Pueyo, Vasilios P. Papanastaris, ElAich Ahmed, and Peter C. de Ruiter. Spatial vegetation patterns and imminent desertification in mediterranean arid ecosystems. *Nature* 449, 213–217 (2007).

Steven J Lade and Thilo Gross. Early warning signals for critical transitions: a generalized modeling approach. *PLoS Computational Biology* 8(2), e1002360 (2012).

Timothy M. Lenton. Early warning of climate tipping points. *Nature Climate Change* 1(4), 201–209 (2011).

Timothy M Lenton, Hermann Held, Elmar Kriegler, Jim W Hall, Wolfgang Lucht, Stefan Rahmstorf, and Hans Joachim Schellnhuber. Tipping elements in the Earth's climate system. *Proceedings of the National Academy of Sciences of the United States of America* 105(6), 1786–93 (2008).

J. Michael T. Thompson and Jan Sieber. Predicting climate tipping as a noisy bifurcation: A review. *International Journal of Bifurcation and Chaos* 21(02), 399–423 (2011).

Mirjam Van Der Mheen, Henk A. Dijkstra, Avi Gozolchiani, Matthijs Den Toom, Qingyi Feng, Jürgen Kurths, and Emilio Hernandez-Garcia. Interaction network based early warning indicators for the atlantic moc collapse. *Geophysical Research Letters* 40(11), 2714–2719 (2013).

Jan Viebahn and Henk A. Dijkstra. Critical transition analysis of the deterministic wind-driven ocean circulation  a flux-based network approach. *International Journal of Bifurcation and Chaos* 24(02), 1430007 (2014).